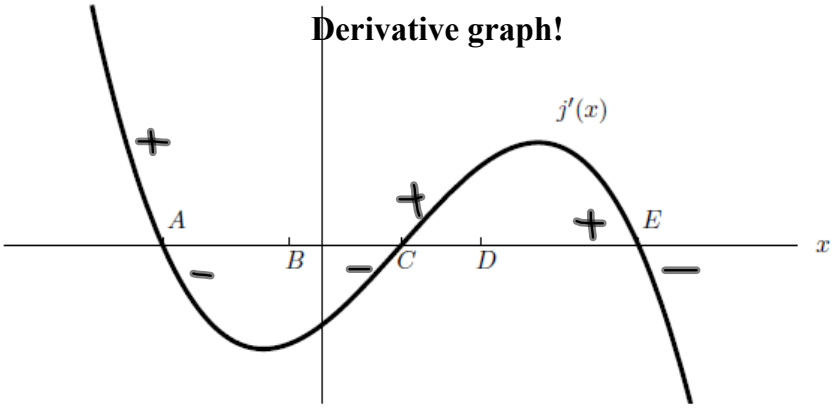
1. [12 points] Consider the graph of $j'(x)$ given here. Note that this is not the graph of $j(x)$.



For each of (a)-(f) below, list **all** x-values labeled on the graph which satisfy the given statement in the blank provided. If the statement is not true at any of the labeled x-values, write "NP". You do not need to show your work. No partial credit will be given on each part of this problem.

(a) The function $j(x)$ has a local minimum at $x =$ C.

(b) The function $j(x)$ has a local maximum at $x =$ A, E.

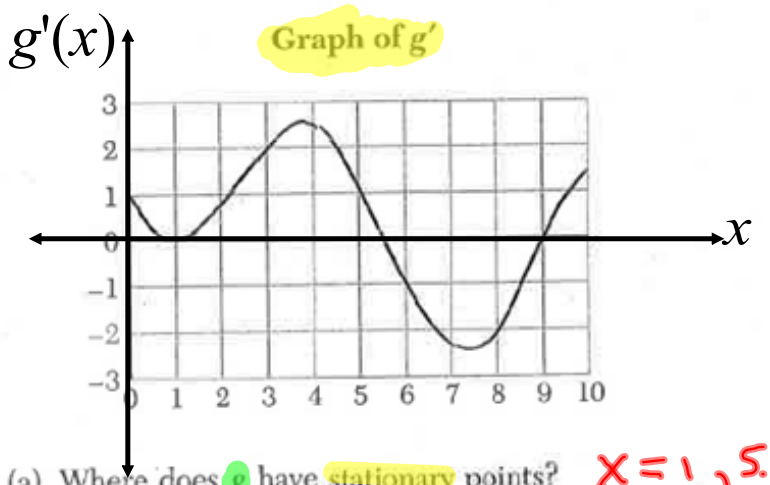
"smiley" (c) The function $j(x)$ is concave up at $x =$ B, C, D.

"frowny" (d) The function $j(x)$ is concave down at $x =$ A, E.

(e) The function $j'(x)$ has a critical point at $x =$ NP.

(f) The function $j''(x)$ is greatest at $x =$ C. → Slope of T.L. is the greatest at P.O.I.

The graph of the derivative of a function g is shown below. Use the graph of g' to answer the following questions about g. [NOTE: The graph of g is not shown.]



(a) Where does g have stationary points? $x = 1, 5.5, 9$

(b) Where does g have local maxima? Local minima? MAX: $x = 5.5$ MIN: $x = 9$

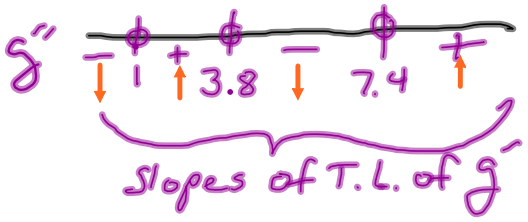
(c) The graph of g' has a local maximum at $x = 3.8$ and a local minimum at $x = 7.4$. What do these facts say about the graph of g? P.O.I.

(d) Is g concave up or concave down at $x = 5$? At $x = 8$? Justify your answers.

Concave down at $x = 5$ b/c the 2nd derivative is negative.

Concave up at $x = 8$ b/c the 2nd derivative is positive.

2nd Derivative Test



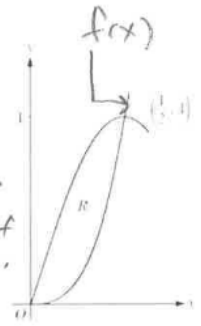
Unit 6 Extra Practice

3. Let R be the region in the first quadrant enclosed by the graphs of $f(x) = 8x^3$ and $g(x) = \sin(\pi x)$, as shown in the figure above.

(a) Write an equation for the line tangent to the graph of f at $x = \frac{1}{2}$.

$f'(x) = 24x^2$ $f'(\frac{1}{2}) = 24(\frac{1}{2})^2 = 24 \cdot \frac{1}{4} = 6$ ← slope of T.L.

$y - 1 = 6(x - \frac{1}{2})$



3. Let f be the function with derivative defined by $f'(x) = x^3 - 4x$. At which of the following values of x does the graph of f have a point of inflection?

(A) 0 (B) $\frac{2}{3}$ (C) $\frac{2}{\sqrt{3}}$ (D) $\frac{4}{3}$ (E) 2

$f''(x) = 3x^2 - 4$

$0 = 3x^2 - 4$

$4 = 3x^2$

$x = \pm\sqrt{\frac{4}{3}} = \pm\frac{2}{\sqrt{3}}$

1. When is the graph of $f(x) = \frac{1}{6}x^3 - \frac{7}{6}x^2 + \frac{5}{2}x^2 - 8x + 12$ concave down?

A) $x < 1$ or $x > \frac{5}{2}$ B) $1 < x < \frac{5}{2}$ C) $x < \frac{1}{2}$ or $x > 5$

D) $\frac{1}{2} < x < 5$ E) The graph is never concave down

$f'(x) = \frac{2}{3}x^3 - \frac{2}{2}x^2 + 5x - 8$

$f''(x) = 2x^2 - 7x + 5$

$0 = (2x - 5)(x - 1)$

$x = \frac{5}{2}$ $x = 1$

		1		$\frac{5}{2}$	
$2x-5$	−		−	0	+
$x-1$	−	0	+		+
	+	0	−	0	+

Down

3. Let g be the function defined by $g(x) = x^4 - 4x^3$. How many relative extrema does g have?

(A) Zero (B) One (C) Two (D) Three

$g'(x) = 4x^3 + 12x^2$

$0 = 4x^2(x + 3)$

$x = 0$ $x = -3$ ← Don't stop here! N.I.A.

		−3		0	
$4x^2$	+		+	0	+
$x+3$	−	0	+		+
	−	0	+	0	+

Only one sign change!